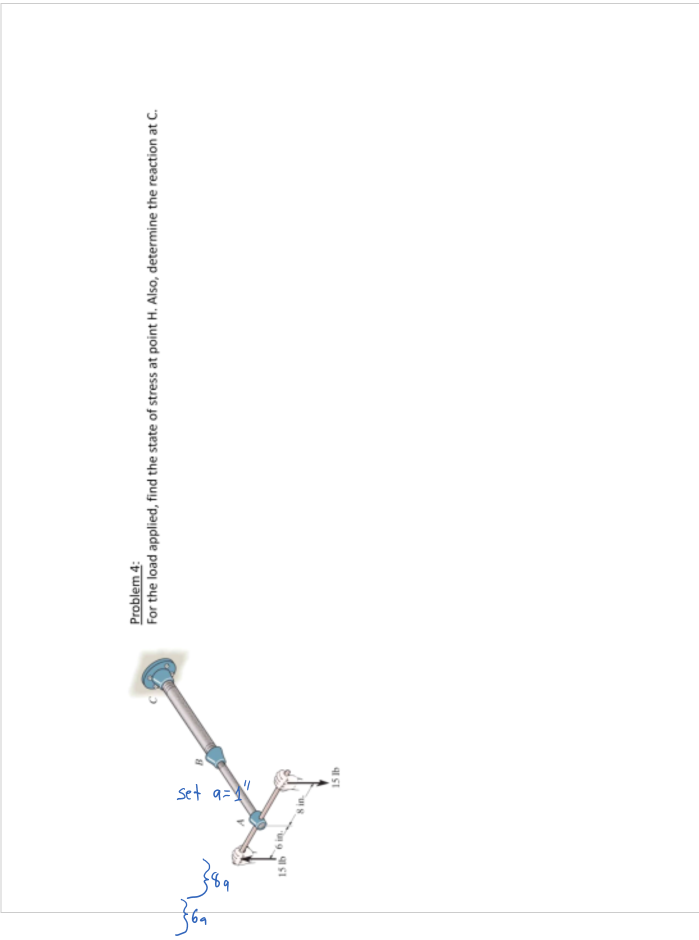
2014ReviewProblems-FinalExam

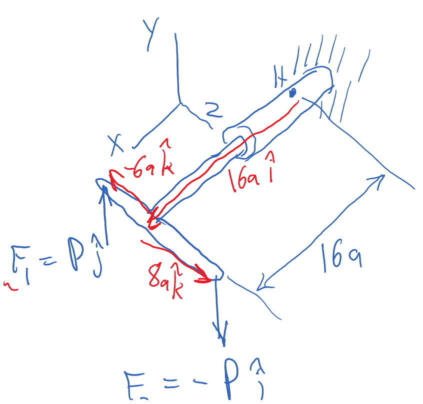
Wednesday, August 4, 2021 9:36 AM

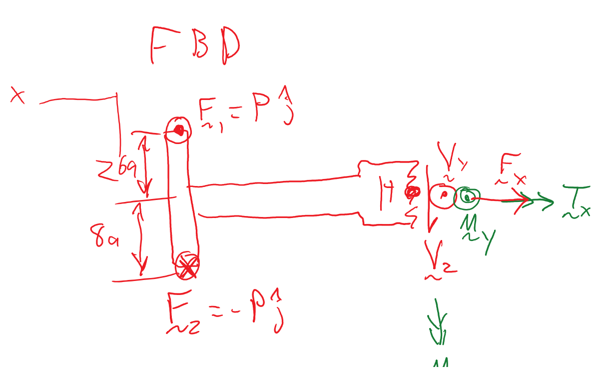
2014ReviewProblems-FinalExam



Problem 4:

For the load applied, find the state of stress at point H. Also, determine the reaction at C.





$\underset{\sim}{F}_2 = -P\hat{j}$

$\underset{\sim}{F}_2 = -P\hat{j}$

$\underset{\sim}{M}_z$

$\underset{\sim}{r}_1 = 16a\,\hat{i} - 6a\,\hat{k}$

$\underset{\sim}{r}_2 = 16a\,\hat{i} + 8a\,\hat{k}$

$$\Sigma\underset{\sim}{F} = \underset{\sim}{0} = \underbrace{-F_x\,\hat{i}} + V_y\,\hat{j} + V_z\,\hat{k} + P\,\hat{j} - P\,\hat{j}$$

$F_x = 0$

$V_z = 0$

$V_y = 0$

$V_y + P - P = 0$

$$\Sigma\underset{\sim}{M} = \underset{\sim}{0} = -T_x\,\hat{i} + M_y\,\hat{j} + M_z\,\hat{k} + \underset{\sim}{r}_1 \times \underset{\sim}{F}_1 + \underset{\sim}{r}_2 \times \underset{\sim}{F}_2$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 16a & 0 & -6a \\ 0 & P & 0 \end{vmatrix} \qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 16a & 0 & 8a \\ 0 & -P & 0 \end{vmatrix}$$

$\hat{i}\,(0 \times 0 - P(-6a))$

$-\hat{j}\cdot(16a \times 0 - 0 \times (-6a))$

$+\hat{k}\cdot(16a\cdot P - 0\cdot 0)$

$\hat{i}\cdot(0\cdot 0 - (-P)(8a))$

$-\hat{j}\cdot(16a\cdot 0 - 0\cdot 8a)$

$+\hat{k}\cdot(16a\cdot(-P) - 0\cdot 0)$

$6Pa\,\hat{i} + 16aP\,\hat{k}$

$8aP\,\hat{i} - 16aP\,\hat{k}$

$$(6aP + 8aP)\,\hat{i} + (16aP - 16aP)\,\hat{k}$$

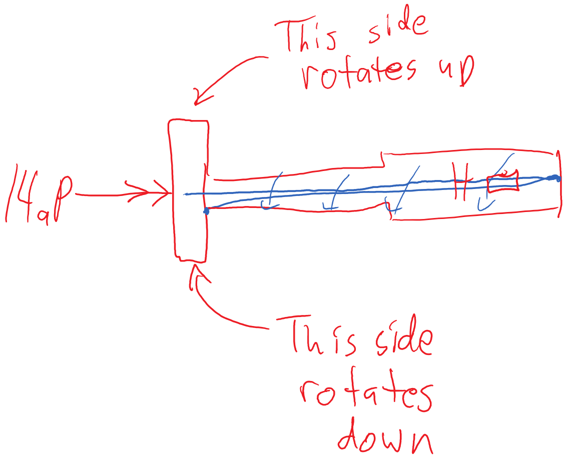
$$14a\cdot P\,\hat{i}$$

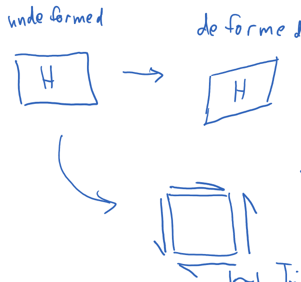
$-T_x\,\hat{i} + 14aP\,\hat{i} = 0\,\hat{i}$

$\left.\begin{array}{l} M_y = 0 \\ M_z = 0 \end{array}\right\}$ No bending!

$T_x = 14\cdot a\cdot P$ Twisting Only!

This side rotates up

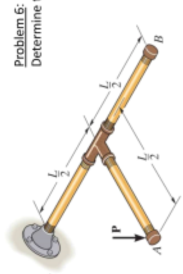




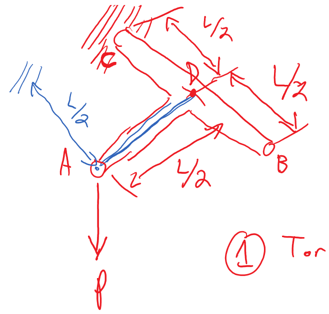
Shear stresses cause the above deformation

$$|\tau| = \frac{T_x \cdot D/2}{J} = \frac{14 \cdot a \cdot P \cdot D}{2 \cdot J}$$

Problem 6:
Determine the vertical displacement of the cross section A. Use deflection table.

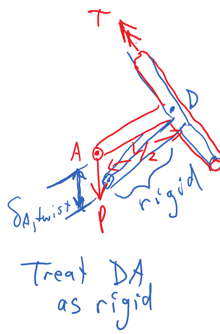


✓① Twisting from C to D
✓② Bending down from C to D
✓③



① Twisting from C to D
② Bending down from C to D
③ Bending down from D to A

① Torque in CD



Treat DA as rigid

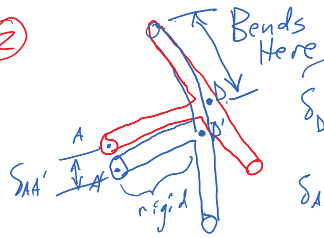
$$T = \frac{-P \cdot L}{2}$$

$$\phi_{CD} = \frac{T \cdot (L/2)}{G \cdot J}$$

Deflection of A will be $-\frac{L}{2}\left(\frac{T \cdot L}{2GJ}\right)$

$$\delta_{A,twist} = -\frac{TL^2}{4GJ} = \frac{-PL^3}{8GJ}$$

②



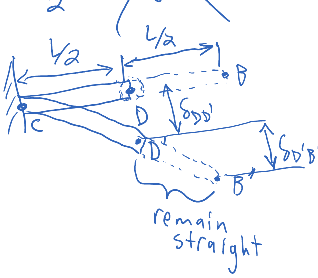
From Table $\left(\delta = \frac{PL^3}{3EI}\right)$ set $L \rightarrow \frac{L}{2}$

$$\delta_{DD'} = \frac{-P \cdot (L/2)^3}{3EI}$$

$$\delta_{AA'} = \delta_{DD'} = \frac{-PL^3}{24EI}$$

Treat
- CD as flexible
- DA as rigid
- DB is unloaded (rigid)

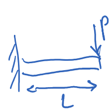
No load in this part



$$\delta_{BB'} = \delta_{DD'} + \overbrace{\theta_D \times \frac{L}{2}}^{\delta_{D'B'}}$$

slope — length from D to B

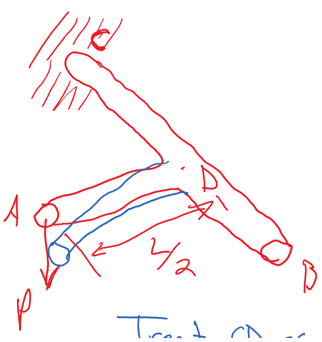
θ_D from table is

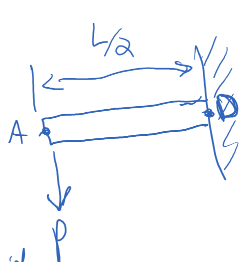


$$\theta_B = \frac{-P \cdot L^2}{2EI} \quad \text{set } L \rightarrow \frac{L}{2}$$

$$\theta_B = \frac{-P(L/2)^2}{2EI}$$

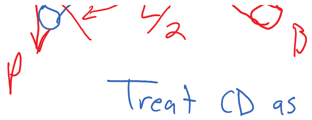
③





Treat CD as rigid

$)^3$



P

Treat CD as rigid
AD as flexible

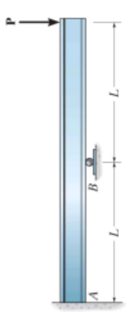
$$\delta_{DA} = \frac{-P(L/2)^3}{3EI} = \frac{-PL^3}{24EI}$$

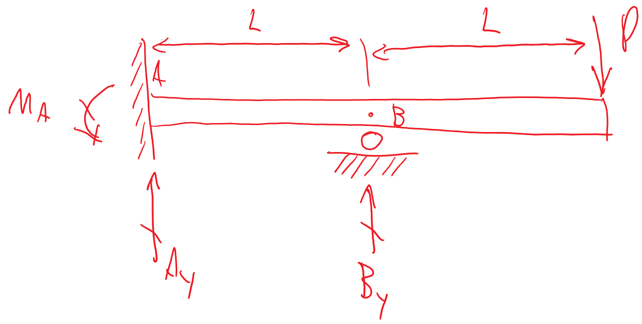
$$\delta_A = \delta_1 + \delta_2 + \delta_3$$

$$\delta_A = \frac{-PL^3}{8GJ} - \frac{PL^3}{24EI} - \frac{PL^3}{24EI}$$

$$\delta_A = -PL^3\left(\frac{1}{8GJ} + \frac{1}{12EI}\right)$$

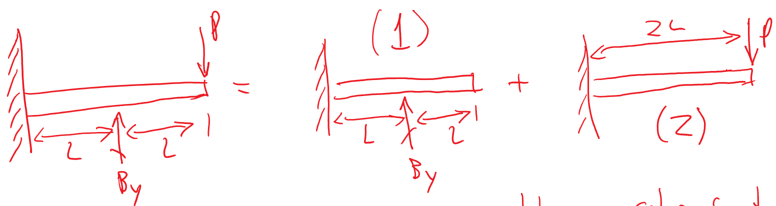
Problem 8:
Determine the reactions at A and B. Use deflection table.





$$(\Sigma M)_A = 0 \Rightarrow M_A + B_y L - 2PL = 0$$

$$(\Sigma F_y) = 0 \Rightarrow A_y + B_y = P$$

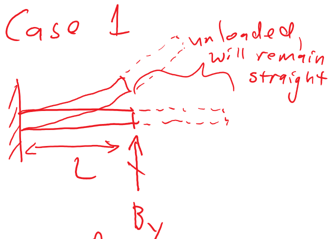


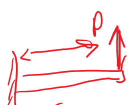
with constraint

$$\underbrace{y(L) = 0}_{\text{geometric compatibility condition}}$$

Goal: $y_{1B} + y_{2B} = y(L) = 0$

Case 1



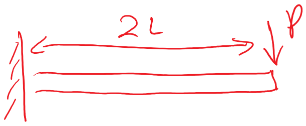


$$\delta_{max} = \frac{PL^3}{3EI}$$

set $P \to B_y$

$$\delta_{1B} = \frac{B_y L^3}{3EI}$$

Case 2



from table

Take $y(x)$ from table

$$y(x) = \frac{-Px^2}{6EI}(3L - x)$$

$$y(x) \; -Px^2 \; (3(2L) - x)$$

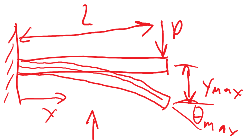
Take $y(x)$ from table
set $L \rightarrow 2L$
and $P \rightarrow P$
Then $x \rightarrow L$

$$y(x) = \frac{-Px^2}{6EI}(3L - x)$$

$$y(x) = \frac{-Px^2}{6EI}(3(2L) - x)$$

$$= \frac{-Px^2}{6EI}(6L - x)$$

$$y(L) = \frac{-PL^2}{6EI}5L = \frac{-5PL^3}{6EI}$$



needed y(half of beam's length)

$$\theta_{max} = \frac{-PL^2}{2EI}$$

$$y_{max} = \frac{-PL^3}{3EI}$$

must use $y(x)$ to find y at halfway point of the beam's length

$$y(x) = \frac{-Px^2}{6EI}(3L - x)$$

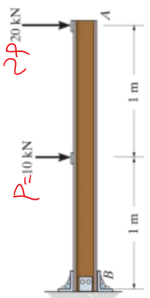
set $x = L$

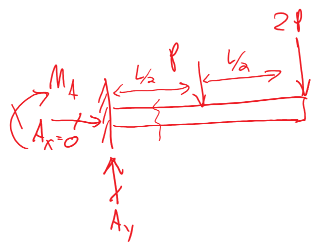
$$\left(\frac{B_y L^3}{3EI} - \frac{5PL^3}{6EI} = 0\right) \times \frac{6EI}{L^3}$$

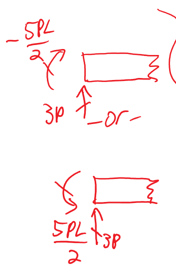
$$2B_y - 5P = 0$$

$$\boxed{B_y = \frac{5P}{2}}$$

Problem 12:
Obtain expressions for the deflection from 0<x<1 and 1<x<2 using ODE. Verify your results by confirming the displacement at A using the superposition method and the deflection table.







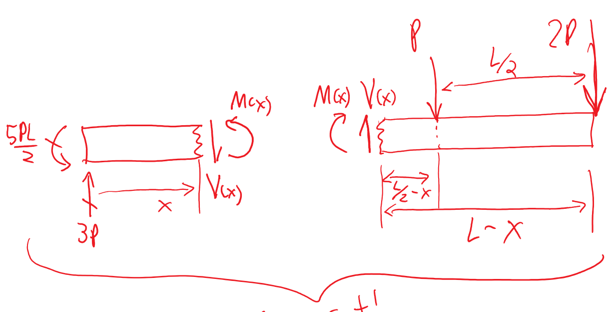
either is fine once the reaction moment is known

$$\Sigma F_y = 0 \Rightarrow -P - 2P + A_y = 0$$

$$A_y = 3P$$

$$(\Sigma M)_A = 0 \Rightarrow -M_A - \frac{PL}{2} - 2P \cdot L = 0$$

$$M_A = -\frac{5PL}{2}$$



same cut!

$\Sigma M_{left} = 0$

$$M(x) + \frac{5PL}{2} - 3Px = 0$$

$$M(x) = 3Px - \frac{5PL}{2}$$

$\Sigma M_{right} = 0$

$$-M(x) - P\left(\frac{L}{2} - x\right) - 2P(L - x) = 0$$

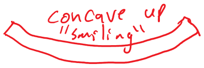
$$M(x) = -\frac{PL}{2} + Px - 2PL + 2Px$$

$$= 3Px - 5PL$$

$M(x) = \ldots$ $= 3Px - \frac{5PL}{2}$

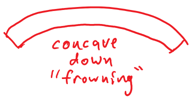
If $M(x) > 0$

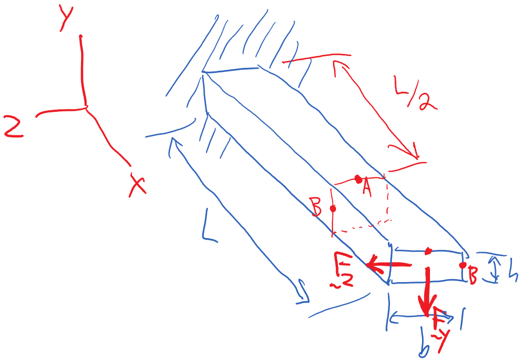
concave up "smiling"



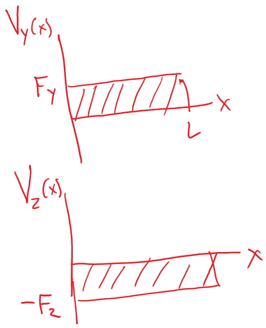
$y''(x) = \frac{M(x)}{EI}$

If $M(x) < 0$





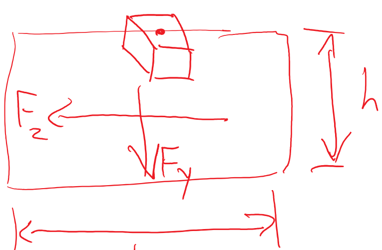
Find state of stress at A & B

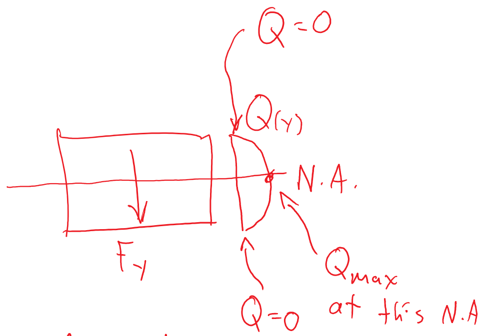


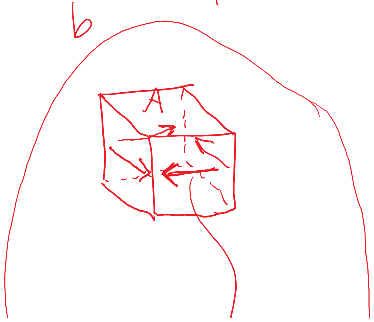
$\underset{\sim}{F}_z = F_z \hat{k}$

$\underset{\sim}{F}_y = -F_y \hat{j}$

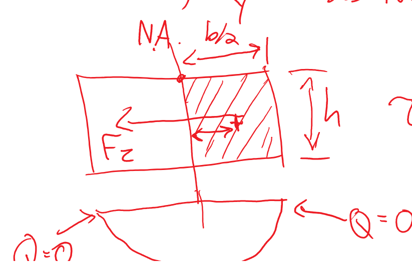
Point A





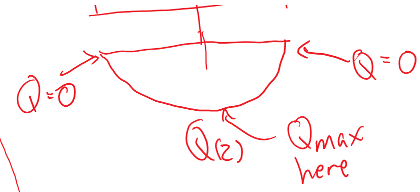


At A, F_y causes NO shear stress

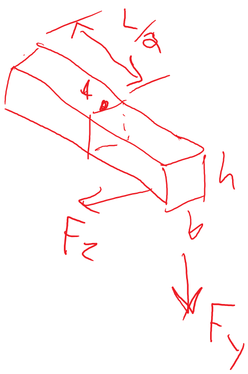


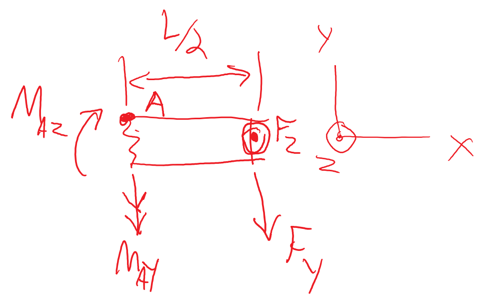
$$\tau_{xz} @ A = \frac{F_z \cdot Q_{max}}{I_y \, t} = \frac{F_z \left(\frac{bh}{2}\right)\left(\frac{b}{4}\right)}{\left(\frac{hb^3}{12}\right) \cdot h}$$

(12)



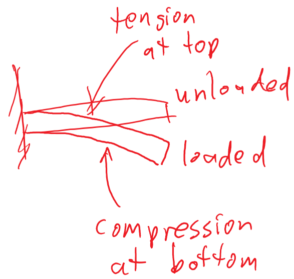
τ_{xz} matches the direction of F_z on this face





$$M_{Az} = -F_y \cdot \frac{L}{2}$$

$$|M_{Ay}| = F_z \cdot \frac{L}{2}$$



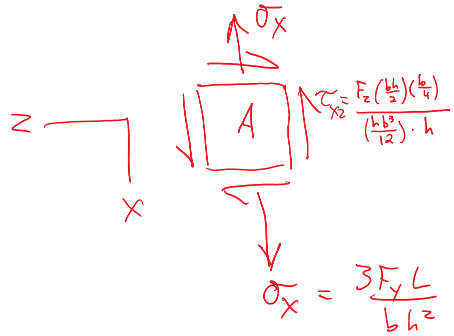
At A, $M_{Az} = \frac{-F_y L}{2}$ causes tension

$$\sigma_x = \frac{-M_{Az} \cdot y_{max}}{I_z}$$

$$\sigma_x = \frac{-\left(\frac{-F_y L}{2}\right)\left(h/2\right)}{\frac{bh^3}{12}} = \frac{3 \cdot F_y \cdot L}{b h^2}$$

At A, $|M_{Ay}| = \frac{F_z \cdot L}{2}$ caused no flexural stress, because A is on the N.A. for the bending moment M_{Ay}

Top View



σ_x
A
z
x
$\tau_{xz} = \dfrac{F_z\left(\frac{bh}{2}\right)\left(\frac{b}{4}\right)}{\left(\frac{hb^3}{12}\right)\cdot h}$
$\sigma_x = \dfrac{3F_y L}{bh^2}$